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|  | **MATHEMATICS METHODS 4**  **SEMESTER 2 2018**  **INVESTIGATION 3**  **The exponential PDF** |

**Marks: 33 Time: 40 minutes**

One continuous probability density function we have encountered is the exponential function. This function is defined as

f(x) =

The exponential function has an mean or expected value E[X] =

Suppose that the random variable X is amount of time a person spends waiting in a bank to be served and is exponentially distributed with a mean of 5 minutes. If E[X] = 5, then = 0.2. Thus if we are asked to determine the probability that a person will need to wait more than 10 minutes to be served in the bank the answer will be

P(X 0.1353

1. [5, 1, 2 marks]

Another way to determine P(X > 10) is to use a cumulative density function.

1. Show the cumulative density function when the mean time is 5 minutes

will be P(X < t) = 1 – e--0.2t and verify that P(X > 10) = 0.1353.

1. Use the c.d.f. (or otherwise) to determine the probability that
2. A person waits in the queue for less than 8 minutes
3. Given a person already waits for at least 10 minutes, will wait for at least 15 minutes.

2. [2 marks]

The median time *m* will be such that 1 – e-0.2m = 0.5. Determine the median time, correct

to 2 d.p.

3. [4 marks]

Determine the mean and median times for the p.d.f.

f(x) =

4. [5 marks]

It would appear that the mean time exceeds the median time in both examples. We can determine the median time by evaluating *m* in terms of .

If we start by saying that 1 – = 0.5, find m in terms of . Explain why the mean will always exceed the median.

5. [1, 1, 1, 2, 2, 2, 3, 2 marks]

Another property of the exponential density function is that it is what we call the **memoryless** property. We can demonstrate this with the following example:

The mean waiting time in a bank queue is 10 minutes.

1. Write the c.d.f. for this distribution.
2. Determine the probability that a person waits at least 5 minutes.

c) Determine the probability that a person waits at least 10 minutes.

d) Given the person waits at least 10 minutes, determine the probability the person waits at least 5 more minutes. [i.e. P(X > 15|X > 10)]

1. Given the person waits at least 15 minutes, determine the probability the person waits at least 5 more minutes.

f) Given the person waits at least 20 minutes, determine the probability the person waits at least 5 more minutes.

g) Comment on your answers, and explain why this is called the memoryless property.

h) We summarise this property as P(X > t + s | X > t) = ? Complete this statement.